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THE METHOD OF MATHEMATICAL INDUCTION IN PROVING IDENTITIES

Mirzakarimova Nigorakhon Mirzakhakimovna
Senior Lecturer, Fergana State University

Kholmatov Akhrorjon Khakimovich
Teacher, Academic Lyceum No. 2 under
Fergana State Technical University

Many problems in mathematics are related to proofs, and such problems can be solved using various methods: elementary methods, using properties, and the method of mathematical induction. Depending on the formulation of the problem, each of these methods may offer different levels of convenience.

In many cases, the method of mathematical induction (MMI) is considered particularly convenient for solving proof-based problems. For example, the use of MMI is effective in proving identities. Below, several examples are presented with solutions obtained both by elementary methods and by the method of mathematical induction.

Example 1. Prove the identity.

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+3\cdot 4\cdot 5+...+n(n+1)(n+2)=\frac{1}{4}n(n+1)(n+2)(n+3)$$

I. Elementary method

First, we express the general term in the form of a difference:

$$k(k+1)(k+2)=\frac{1}{4}[k(k+1)(k+2)(k+3)-(k-1)]$$

By simplifying this difference, we obtain the general term given above.

Now, let us denote the given sum by Sn and write each term of this sum in the form of a difference. In this sum, the middle terms cancel each other out, leaving only the second term of the first bracket and the first term of the last bracket. That is,

$$S_n = \frac{1}{4}[n(n+1)(n+2)(n+3)-0]$$

$$S_n = \frac{n(n+1)(n+2)(n+3)}{4}$$

Thus, the identity is proved.

II. Solution using the Method of Mathematical Induction

1. First, we verify that the identity holds for $n=1$:

$$S_{ch} = 1\cdot 2\cdot 3 = 6$$

$$S_{0'} = \frac{1\cdot(1+1)\cdot(1+2)\cdot(1+3)}{4} = \frac{1\cdot 2\cdot 3\cdot 4}{4} = 6$$

$$S_{ch} = S_{0'}$$

2. Assume that the given identity is true for $n=k$:

$$1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + 3\cdot 4\cdot 5 + ... + k(k+1)(k+2) = \frac{1}{4}k(k+1)(k+2)(k+3)$$

3. Using this assumption, we show that the equality holds for an arbitrary $k+1 \in N$

We need to obtain:

$$1\cdot 2\cdot 3 + 2\cdot 3\cdot 4 + ... + k(k+1)(k+2) + (k+1)(k+2)(k+3) =$$

$$= \frac{1}{4}k(k+1)(k+2)(k+3)(k+4)$$

According to the assumption,

$$1\cdot 2\cdot 3+2\cdot 3\cdot 4+...+k(k+1)(k+2)+(k+1)(k+2)(k+3)=$$

$$=\frac{1}{4}k(k+1)(k+2)(k+3)=\frac{1}{4}(k+1)(k+2)(k+3)(k+4)$$

Hence, the given equality holds for $\forall n\in N$. Therefore, the identity is true for all natural numbers nnn.

Example 2. Prove the identity. $1^3+2^3+3^3+...+n^3=(1+2+3+...+n)^2$

I. Elementary method

1. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$$\left.\begin{aligned}
&1^3=(1+0)^3-0^3+3\cdot 0^2\cdot 1+3\cdot 0\cdot 1^2+1^3\\
&2^3=(1+1)^3=1^3+3\cdot 1^2\cdot 1+3\cdot 1\cdot 1^2+1^3\\
&3^3=(2+1)^3=2^3+3\cdot 2^2\cdot 1+3\cdot 2\cdot 1^2+1^3\\
&4^3=(3+1)^3=3^3+3\cdot 3^2\cdot 1+3\cdot 3\cdot 1^2+1^3\\
&-----------------------\\
&(n+1)^3=(n+1)^3=n^3+3\cdot n^2\cdot 1+3\cdot n\cdot 1^2+1^3
\end{aligned}\right| +$$

$$1^3+2^3+3^3+...+n^3+(n+1)^3=1^3+2^3+...+n^3+3\cdot(1^2+2^2+...+n^2)+$$
$$+3\cdot(1+2+...+n)+(n+1)\cdot 1$$

$$(n+1)^3=3\cdot(1^2+2^2+...+n^2)+3\cdot\frac{1+n}{2}\cdot n+(n+1)$$

$$3\cdot(1^2+2^2+...+n^2)=(n+1)\left((n+1)^2-\frac{3n}{2}-1\right)$$

$$3\cdot(1^2+2^2+...+n^2)=(n+1)\frac{2n^2+4n+\cancel{2}-3n-\cancel{2}}{2}$$

$$1^2+2^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$$

We make use of the last equality.

2. 2) $(a+b)^4=a^4+4a^3\cdot b+6\cdot a^2b^2+4\cdot a\cdot b^3+b^4$

$$\left.\begin{aligned}
&1^4=(0+1)^4=0^4+4\cdot 0^3\cdot 1+6\cdot 0^2\cdot 1^2+4\cdot 0\cdot 1^3+1^4\\
&2^4=(1+1)^4=1^4+4\cdot 1^3\cdot 1+6\cdot 1^2\cdot 1^2+4\cdot 1\cdot 1^3+1^4\\
&3^4=(2+1)^4=2^4+4\cdot 2^3\cdot 1+6\cdot 2^2\cdot 1^2+4\cdot 2\cdot 1^3+1^4\\
&4^4=(3+1)^4=3^4+4\cdot 3^3\cdot 1+6\cdot 3^2\cdot 1^2+4\cdot 3\cdot 1^3+1^4\\
&-----------------------------\\
&(n+1)^4=(n+1)^4=n^4+4\cdot n^3\cdot 1+6\cdot n^2\cdot 1^2+4\cdot n\cdot 1^3+1^4
\end{aligned}\right| +$$

$$1^4+2^4+...+(n+1)^4=1^4+2^4+...+n^4+4(1^3+2^3+...+n^3)+$$

$$+6\cdot(1^2+2^2+...+n^2)+4(1+2+...+n)+(n+1)\cdot 1$$

$$(n+1)^4=4\cdot(1^3+2^3+...+n^3)+6\cdot(1^2+2^2+...+n^2)+4(1+2+...+n)+(n+1)$$

$$4(1^3+2^3+...+n^3)=(n+1)^4-6\cdot\frac{n(n+1)(2n+1)}{6}-4\cdot\frac{1+n}{2}\cdot n-(n+1)$$

$$4(1^3+2^3+...+n^3)=(n+1)\left((n+1)^3-n(2n+1)-2\cdot n-1\right)$$

$$4(1^3+2^3+...+n^3)=(n+1)\left((n+1)^3-2n^2-3n-1\right)$$

$$4(1^3+2^3+...+n^3)=(n+1)\left(n^3+3n^2+3\cancel{n}+\cancel{1}-2n^2-3\cancel{n}-\cancel{1}\right)$$

$$1^3+...+n^3=\frac{(n+1)(n+1)}{4}$$

$$1^3+...+n^3=(1+2+...+n)^2$$

Thus, the identity is proved.

II. Proof using the Method of Mathematical Induction

1. We verify that the given equality is true for $n=1$

$1^3 = 1^2$

$$1=1$$

2. Assume that the given equality holds for $n=k$:

$1^3+2^3+3^3+...+k^3=(1+2+3+...+k)^2$

3. We show that the equality holds for an arbitrary $k+1 \in N$.

$1^3+2^3+3^3+...+k^3+(k+1)^3=(1+2+3+...+k+k+1)^2$

Using the assumption, we obtain:

$(1+2+3+...+k)^2+(k+1)^3=(1+2+3+...+k+(k+1))^2$

$$\left(\frac{1+k}{2}\cdot k\right)^2+(k+1)^3=(1+k)^2\left(\frac{k^2}{4}+k+1\right)=$$

$$=\frac{1}{4}(1+k)^2\cdot(k^2+4k+4)=\frac{1}{4}(1+k)^2(k+2)^2=$$

$$=\left(\frac{(k+1)(k+2)}{2}\right)^2=(1+2+3+...+k+(k+1))^2$$

Thus, the given equality is an identity for $\forall n \in N$.

From these examples, it can be seen that for proving such identities, the method of mathematical induction is more effective than elementary methods.

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